SECTION 1 Multiple Choice Question (MCQ)

- This section contains **TEN (10)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, darken the bubble corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: $+3$	If **ONLY** the bubble corresponding to the correct option is darkened.
Zero Marks	: 0	If none of the bubble is darkened (i.e., the question is unanswered).
Negative Marks	: -0.75	In all other cases.

Q.1 The scores of ten students out of eleven are $3, 4, 6, 7, 8, 9, 9, 10, 10, 10$. If the score of the eleventh student can be any integer from 0 to 10, then what is the probability that the median of these scores is 8?

(A) $\frac{6}{11}$ (B) $\frac{7}{11}$ (C) $\frac{8}{11}$ (D) $\frac{9}{11}$

Q.2 How many words (with or without meaning) can be formed by shuffling the letters of the word "ABUDHABI" such that the word starts with a vowel and every vowel is followed by a consonant?

(A) 36 (B) 72 (C) 144 (D) 576

Q.3 A pair of opposite sides of a parallelogram are perpendicular to a diagonal. If the ratio of the lengths of a shorter side to the shorter diagonal is $3 : 4$, then what is the ratio of the lengths of a longer side to the shorter diagonal?

(A) $4 : 3$ (B) $5 : 3$ (C) $5 : 4$ (D) $7 : 5$

Q.4 The value of the sum

$$\sum_{n=1}^{10}(3n^2 + 2n - 1)$$

is

(A) 1255 (B) 1264 (C) 1265 (D) 760

Q.5 Let $A = \begin{pmatrix} -1 & 1+\lambda \\ 1+\bar{\lambda} & -1 \end{pmatrix}$, where λ is a complex number and $\bar{\lambda}$ is the complex conjugate of λ. If the determinant of A is a positive real number, then the real part of λ is

(A) positive (B) negative (C) zero (D) one

Q.6 Which one of the following is the area of the region enclosed by the curves $y = x^2$ and $y = \sqrt{x}$?

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Q.7 Let $\mathbb{N}$ be the set of natural numbers and let $f: \mathbb{N} \to \mathbb{N}$ be defined by $f(n) = 4n^2 + 8n + 2024$. Then f is

(A) neither one-one nor onto
(B) one-one but NOT onto
(C) onto but NOT one-one
(D) both one-one and onto

Q.8 Let $a > 0$ be a real number and let $f(x)$ be the quadratic polynomial such that $f(a) = f(-a) = 0$ and $f(0) = -a^2$. Then the value of

$$\lim_{x \to a} \frac{f(x)}{\sin(\sin(x - a))}$$

is

(A) 0 (B) not defined (C) $2a$ (D) $-2a$

Q.9 Which one of the following points lies on the normal to the ellipse $x^2 + 4y^2 = 4$ at the point $\left(1, \frac{\sqrt{3}}{2}\right)$?

(A) $\left(\frac{3}{4}, 0\right)$ (B) $\left(\frac{1}{2}, 0\right)$ (C) $\left(0, -\frac{3}{4}\right)$ (D) $\left(0, -\frac{1}{2}\right)$

Q.10 Let

$$f(x) = \begin{cases} 3x^2 + 2x, & 0 \le x \le 1 \\ 4x, & 1 < x \le 2 \end{cases}$$

and let $g(x) = \int_0^x f(t)\, dt$. Then which one of the following statements is true?

(A) g is differentiable at $x = 1$

(B) $\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = 4$

(C) $\lim_{x \to 1^-} \frac{g(x) - g(1)}{x - 1} = 5$ and $\lim_{x \to 1^+} \frac{g(x) - g(1)}{x - 1} = 4$

(D) $\lim_{x \to 1^-} \frac{g(x) - g(1)}{x - 1} = 3$ and $\lim_{x \to 1^+} \frac{g(x) - g(1)}{x - 1} = 4$

SECTION 2 SINGLE DIGIT NON-NEGATIVE INTEGER (SDI)

- This section contains **TEN (10)** questions.
- The answer to each question is a **SINGLE DIGIT NON-NEGATIVE INTEGER (SDI)**.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 In all other cases.

Q.11 The value of the sum

$$\sum_{n=1}^{99999} \log_{10}\left(\frac{n+1}{n}\right)$$

is _____

Q.12 In a club of 50 members, each member speaks at least one of Arabic, English, and French. Each language is spoken by exactly 40 members and any two of these languages are spoken by exactly 30 members. If all the three languages are spoken by exactly N members, then the value of $\frac{N}{5}$ is _____

Q.13 The faces of a die are marked with $1, 2, 2, 5, 5, 6$, and it is rolled twice. If p is the probability that the sum of the outcomes is 7, then the value of $18p$ is _____

Q.14 Three red coins numbered 1 to 3, and three blue coins numbered 1 to 3 are to be arranged in a row such that the first and the last coins are both blue. If N is the number of different arrangements, then the value of $\frac{N}{18}$ is _____

Q.15 The number of values of $x \in [0, 2\pi]$ satisfying the equation

$$2\sin 2x + 2 = 8\cos x + \sin x$$

is _____

Q.16 Let $\alpha = \int_0^1 \frac{1}{x^2+3x+2}\, dx$. Then the value of $3e^{\alpha}$ is _____

Q.17 Let $A = \begin{pmatrix} x^2 & 2 & y \\ 0 & -1 & 2 \\ x & 0 & x \end{pmatrix}$, where x is a real number satisfying $x^2 + x = 2$ and y is a positive real number. If the determinant of A is 4, then the value of $x^2 + 2y$ is _____

Q.18 The number of real values of x satisfying $2\cos\big(x(x+2)\big) = 2^x + 2^{-x}$ is _____

Q.19 A solid hemisphere of radius 21cm is melted to form a solid cylinder so that the ratio of the curved surface area to the total surface area of the cylinder is $2:5$. If r is the radius, in cm, of the base of the cylinder, then the value of $\frac{r}{7}$ is _____

Q.20 Suppose $i = \sqrt{-1}$ and $\overline{w}$ is the complex conjugate of the complex number w. Let $z = x + iy$ and $\bar{z} = (p + i\,q)^3$, where $x, y, p\ (\neq 0)$ and $q\ (\neq 0)$ are real numbers. Then the value of

$$\frac{-1}{p^2+q^2}\left(\frac{x}{p}+\frac{y}{q}\right)$$

is _____

SECTION 3: Paragraph based MCQ

- This section contains **FIVE (05)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : $+3$ If **ONLY** the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 If none of the bubble is darkened (i.e., the question is unanswered).
  Negative Marks : -0.75 In all other cases.

PARAGRAPH I

$PQRS$ is a parallelogram, where P is the origin, and the position vectors of Q and S are $\hat{\imath}+\hat{\jmath}$ and $\hat{\jmath}+\hat{k}$, respectively.

Q.21 What is the volume of the parallelepiped formed by $PQRS$ and a unit vector perpendicular to $PQRS$ which starts from P?

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$

PARAGRAPH I

$PQRS$ is a parallelogram, where P is the origin, and the position vectors of Q and S are $\hat{\imath}+\hat{\jmath}$ and $\hat{\jmath}+\hat{k}$, respectively.

Q.22 What is the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$?

(A) 90^o (B) 60^o (C) 45^o (D) 30^o

PARAGRAPH II

Consider the line $L: 4x-3y=12$.

Q.23 Which one of the following lines is perpendicular to L and is at a distance 5 from the origin?

(A) $3x+4y=25$ (B) $3x+4y=12$
(C) $-3x+4y=25$ (D) $-3x+4y=12$

PARAGRAPH II

Consider the line $L : 4x - 3y = 12$.

Q.24 What is the radius of the circle which passes through the origin, the point of intersection of L with the x-axis, and the point of intersection of L with the y-axis?

(A) 5 (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$

PARAGRAPH III

Let $p(x) = x^2 + bx + c$, where b and c are positive real numbers.

Q.25 Which one of the following conditions ensures that $p(x) > 0$ for every real value of x?

(A) $b = c$
(B) $b < 4c$
(C) $b^2 > 4c$
(D) $b^2 < c$

PARAGRAPH III

Let $p(x) = x^2 + bx + c$, where b and c are positive real numbers.

Q.26 If α is a nonzero real number such that $\alpha \neq \frac{1}{\alpha}$, and both α and $\frac{1}{\alpha}$ are the roots of $p(x) = 0$, then which one of the following statements is true?

(A) $b = 1$
(B) $b = 2$
(C) $c = 1$
(D) $c = 2$

PARAGRAPH IV

Consider the complex numbers $z = 1 + i$ and $w = 1 - i$, where $i = \sqrt{-1}$. Suppose O denotes the origin and Q denotes the point represented by z on the complex plane.

Q.27 The complex number $\frac{z^{100}}{w^{80}}$ is equal to

(A) 512 (B) -512 (C) 1024 (D) -1024

PARAGRAPH IV

Consider the complex numbers $z = 1 + i$ and $w = 1 - i$, where $i = \sqrt{-1}$. Suppose O denotes the origin and Q denotes the point represented by z on the complex plane.

Q.28 Suppose $OPQR$ is the rhombus whose area is 4 and the point R lies in the second quadrant. Which one of the following complex numbers represents the point R?

(A) $-\frac{3}{2}+\frac{5}{2}i$ (B) $-3+\frac{5}{2}i$ (C) $-\frac{3}{2}+5i$ (D) $-3+5i$

PARAGRAPH V

Let $f(x) = 2x^3 - 6x^2 + 24x$ for every real number x, and

$$g(x) = \begin{cases} \max\{f(t): 0 \le t \le x\}, & 0 \le x \le 1 \\ 5x + 15, & x > 1. \end{cases}$$

Q.29 Which one of the following statements is true?

(A) g is NOT continuous at $x = 1$
(B) g is continuous at $x = 1$ but NOT differentiable at $x = 1$
(C) $g(f(x)) = f(f(x))$ for all $0 \le x \le 1$
(D) g is differentiable at $x = 1$

PARAGRAPH V

Let $f(x) = 2x^3 - 6x^2 + 24x$ for every real number x, and

$$g(x) = \begin{cases} \max\{f(t): 0 \le t \le x\}, & 0 \le x \le 1 \\ 5x + 15, & x > 1. \end{cases}$$

Q.30 Which one of the following statements is true?

(A) f is increasing but g is NOT increasing
(B) f is NOT increasing but g is increasing
(C) both f and g are increasing
(D) g attains both maximum and minimum values in $[0, \infty)$